CONVERTING DATES INTO THE METONIC AND KALLIPPIC CALENDARS

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In a seminal article on the link between the Greek astronomical and the Athenian civil calendars, B. L. Van der Waerden converted the dates of several ancient astronomical observations into the calendars of Euktemon-Meton and Kallippus.[1] He took 13 Skirophorion = 21 Phamenoth in the archonship of Apseudes (27 June 432 BC)[2] and 1 Hekatombaion = 28 June 330 BC as the epochs of these two astronomical calendars respectively and computed the current month and day numbers for each date. I have formulated his prescribed procedure as follows:

The equivalent number of calendar days D in M months is:[3]

$$D = 30M - \left\lfloor \frac{30M}{64} \right\rfloor \qquad \ldots\ldots\ldots \qquad \text{Equ. 1}$$

Conversely, the equivalent number of elapsed Metonic or Kallippic months M and remaining days R in a given number of elapsed days D from the above quoted two epochs are:

$$M = \left\lfloor \frac{1}{30}\left(D + \left\lfloor \frac{D}{63} \right\rfloor + d\right) \right\rfloor \qquad \ldots\ldots\ldots \qquad \text{Equ. 2}$$

$$R = \left(D + \left\lfloor \frac{D}{63} \right\rfloor + d\right) - 30M \qquad \ldots\ldots\ldots \qquad \text{Equ. 3}$$

where d = 12 and d = 0 for the Metonic and Kallippic calendars respectively.

Taking Hekatombaion as the first calendar month, the quotient Q from Equ. 2 gives the current month-number M_c in the Metonic calendar while $M_c = Q + 1$ in the Kallippic calendar. The current day-number $D_c = R + 1$ is the same for both calendars.

However, Van der Waerden gave no reason for the 1-day discrepancy in some day-numbers in the month that immediately follows a set of 30-30 months. In describing the organisation of the Metonic cycle, Geminus (*Eisagoge*, 8.52) reveals the occurrence of such sets:

> *Ἐν δὲ τοῖς σλε´ μησὶ κοίλους ἔταξαν ρί, πλήρεις δὲ ρκέ, ὥστε μὴ ἄγεσθαι ἕνα καὶ ἕνα κοῖλον καὶ πλήρη, ἀλλὰ καὶ δύο ποτὲ κατὰ τὸ ἑξῆ πλήρεις.*[4]
>
> Within the 235 months, they made 110 hollow and 125 full, so that hollow and full months did not always follow one another alternately, but sometimes there would be two full months in succession.[5]

This is now confirmed by the recurrent sets of 30-30 months in both the Metonic and Kallippic calendars discussed below.[6]

To demonstrate this and explain the reason for the 1-day discrepancy we must first identify the Metonic and Kallippic full and hollow months and their correct sequence. This could be done by counting from the epochs of the two cycles and, as prescribed by Geminus, dropping a day after every 63rd day.[7]

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1 Van der Waerden (1960), 170-174.
2 Fotheringham (1924), 383; Samuel (1972), 44-45.
3 The brackets ⌊ ….⌋ signify the integer part of the quotients, i.e., rounded down to the nearest integer.
4 Manitius (1898), 120-121; Samuel (1972), 43; Aujac (1975), 56-57.
5 Heath (1913), 294.
6 With one 29-29 case in the Kallippic calendar, probably the last two months of the cycle.
7 Manitius (1898), 122-123; Heath (1913), 294; Samuel (1972), 43; Aujac (1975), 57.

Table 1 below sets out the Metonic months, using Fotheringham's proposed intercalation in years 2, 5, 8, 10, 13, 16, and 18 of the cycle.[8] This reveals 15 cases of 30-30 months whose positions in the calendar may be determined from the following equations:

$$N_1 = 9 + 15\left\lfloor \frac{n-1}{2} \right\rfloor + 17\left\lfloor \frac{n}{2} \right\rfloor \quad \text{.........} \quad \text{Equ. 4}$$

$$N_2 = N_1 + 1 \quad \text{.........} \quad \text{Equ. 5}$$

Where N_1 and N_2 are month numbers and $1 \le n \le 15$ is the set number of each 30-30 month pair.

The following examples demonstrate the discrepancy in some current day-numbers that fall in the month that comes immediately after a set of 30-30 months:

Example 1: Counting from the epoch of the cycle, convert 7261 elapsed days into the calendar of Euktemon-Meton:

Since 7261 > 6940, the quotient $Q_1 = 1$ and remainder $R_1 = 321$ from 7261 ÷ 6940 give the numbers of elapsed cycles and days in the current cycle respectively. From $R_1 \div 63$ we get $Q_2 = 5$ and T = 321 + 5 + 12 = 338. Dividing T by 30 gives $Q_3 = 11$ and $R_2 = 8$. Using equations 2 and 3, we get $M_c = Q_3 = 11$ as the current month-number, following immediately the first 30-30 month pair 9-10 (n = 1 in Equ. 4), and $D_c = R_2 + 1 = 9$ as the current day-number. However, counting from 13 Skirophorion in the current cycle, we have 17 + 6 × 30 + 4 × 29 = 313 days up to the end of month 10. This places the 321st day in day 8 of the 11th month, i.e., one day earlier than the day-number from Equ. 3 above.

To explain the discrepancy, we must determine the position of the eliminated day (P_e) in the 11th Metonic (hollow) month. This may be obtained from the following Equations:

$$N_e = \left\lfloor \frac{30M + 18}{64} \right\rfloor \quad \text{.........} \quad \text{Equ. 6}$$

$$P_e = 64N_e - [18 + 30(M - 1)] \quad \text{.........} \quad \text{Equ. 7}$$

Where M = month number.

From these we get $N_e = 5$ for $M = M_c = 11$ and $P_e = 64 \times 5 - (18 + 30 \times 10) = 2$, the position of the eliminated day in the hollow month 11.

Since day 2 of the 11th month is eliminated, the count goes from 1 to 3 and so pushes the 8th day forward by 1 to day 9 of month. We can check this by the following two examples:

Example 2: Counting from the epoch of the cycle, convert 7255 elapsed days into the calendar of Euktemon-Meton:

Since 7255 > 6940, the quotient $Q_1 = 1$ and remainder $R_1 = 315$ from 7255 ÷ 6940 give the numbers of elapsed cycles and days in the current cycle respectively. From $R_1 \div 63$ we get $Q_2 = 5$ and T = 315 + 5 + 12 = 332. Dividing T by 30 gives $Q_3 = 11$ and $R_2 = 2$. Using equations 2 and 3, we get $M_c = Q_3 = 11$ as the current month-number, following immediately the 30-30 month pair 9-10, and $D_c = R_2 + 1 = 3$ as the current day-number. However, counting from 13 Skirophorion in the current cycle, we have 17 + 6 × 30 + 4 × 29 = 313 days up to the end of month 10. This

[8] Fotheringham (1924), 387. Van der Waerden (1960), 175-177, argues that years 2 or 3, 10 or 11, and 18 or 19 could have been intercalary. In any case, the disposition of the embolismic years does not affect the sequence of full and hollow months. In a future note on Seleucid time-reckoning I will demonstrate that the Macedonian equivalent of the Metonic calendar intercalated a month in year 19 of the cycle.

places the 315th day in the 2nd day of the 11th month, i.e., one day before day-number 3 from Equ. 3 above. As shown earlier, because day 2 of the 11th month is omitted, the count goes from 1 to 3. The following example further clarifies the situation:

Example 3: Counting from the epoch of the cycle, convert 7254 elapsed days into the calendar of Euktemon-Meton:

Since 7254 > 6940, the quotient $Q_1 = 1$ and remainder $R_1 = 314$ from 7254 ÷ 6940 give the numbers of elapsed cycles and days in the current cycle respectively. Since this is 1 day less than the previous figure 7255 days, one expects to get the current day number as 3 – 1 = 2. However, from $R_1 \div 63$ we get $Q_2 = 4$ and T = 314 + 4 + 12 = 330. Dividing T by 30 gives $Q_3 = 11$ and $R_2 = 0$. Using equations 2 and 3, we get $M_c = Q_3 = 11$ as the current month-number, following immediately the 30-30 month pair 9-10, and $D_c = R_2 + 1 = 1$ as the current day-number. This time, counting from 13 Skirophorion in the current cycle, we get 17 + 6 × 30 + 4 × 29 = 313 days up to the end of month 10. Our 314th day thus becomes the 1st day of the 11th month, i.e., no earlier than the day-number from Equ. 3 above. This is because day 1 of the 11th month falls before the omitted day and so remains unaffected by the elimination of the 2nd day.

I should point out that this 1-day discrepancy occurs irrespective of both the epoch-date of the cycle and the length of Skirophorion in 432 BC. For example, if the latter is considered hollow, the month-numbers in the 30-30 sets may be obtained from the following equations without affecting the 1-day difference:

$$N_1 = 9 + 15\left\lfloor \frac{n}{2} \right\rfloor + 17\left\lfloor \frac{n-1}{2} \right\rfloor \qquad \ldots\ldots\ldots \qquad \text{Equ. 8}$$

$$N_2 = N_1 + 1 \qquad \ldots\ldots\ldots \qquad \text{Equ. 9}$$

If, on the other hand, the cycle begins with 1 Hekatombaion 432 BC, the below equations determine the positions of its 30-30 months, again with no effect on the 1-day discrepancy:

$$N_1 = 1 + 15\left\lfloor \frac{n}{2} \right\rfloor + 17\left\lfloor \frac{n-1}{2} \right\rfloor \qquad \ldots\ldots\ldots \qquad \text{Equ. 10}$$

$$N_2 = N_1 + 1 \qquad \ldots\ldots\ldots \qquad \text{Equ. 11}$$

Equations 10 and 11 may be used to find the 30-30 sets of months in the Kallippic calendar also, beginning on 1 Hekatombaion 330 BC (28 June) with $1 \le n \le 59$.

The position of the eliminated day (P_e) in month M of the Kallippic calendar may be obtained for the following equations:

$$N_e = \left\lfloor \frac{30M}{64} \right\rfloor \qquad \ldots\ldots\ldots \qquad \text{Equ. 12}$$

$$P_e = 64N_e - 30(M - 1) \qquad \ldots\ldots\ldots \qquad \text{Equ. 13}$$

In the absence of lists of calendar months similar to those in Tables 1 and 2 below, equations 1-13 allow a speedy reconstruction of the sequence of Metonic and Kallippic months. Apart from the 30-30 sets, the remainder are alternating 30-29 months, save the last month of the Kallippic cycle which is taken to be hollow, not full.[9]

Unfortunately, we have no ancient astronomical records dated specifically on the Metonic calendar with day, month and year numbers. Fotheringham considered three lunar eclipses cited

[9] This is assumed by Fotheringham (1924), 389. On the elimination of the 441st day in the Kallippic cycle cf. Jones (2000), 154, who proposes removal of the 63rd day at 24 points of the cycle. However, for the sake of simplicity, it is possible that elimination followed the pattern adduced by Fotheringham.

by Ptolemy in the Almagest and concluded that Hipparchus (*ca*. 190-120 BC) had mapped their original Babylonian dates on to the Metonic calendar with Attic archons and months.[10] However, since these converted dates lack day-numbers, it is impossible to confirm Fotheringham and plead that Hipparchus indeed translated the Babylonian dates into the Metonic calendar. As three later examples from the Seleucid period indicate, Hipparchus may have employed the Kallippic calendar in converting the Babylonian lunisolar dates.[11] The Seleucid examples are, according to Ptolemy, κατὰ Χαλδαίος, that is, on the Chaldean/Babylonian calendar. They appear in the Almagest and are dated 5 Apellaios 67 SEB, 14 Dios 75 SEB, and 5 Xandikos 82 SEB.[12] The first two relate to the early morning observations of Mercury on 19 Nov. 245 BC and 30 Oct. 237 BC respectively. The third concerns an evening observation of Saturn on 1 Mar. 229 BC. Given that the original lunisolar dates of these observations would have had little practical application for non-Babylonian astronomers, they must have been subsequently mapped on to an astronomical calendar. Simple computations show that these particular Chaldean dates conform to a Kallippic calendar that started off with midnight 1 Hyperberetaios on 27 August 311 BC (JDN = 1608069), i.e. a Kallippic version of the Seleucid era on the Babylonian count.[13]

As for the Kallippic calendar, Fotheringham used four stellar occultations by the moon that Timocharis had observed and found that they had a common epoch.[14] These are dated 25 Poseideon, year 36 of the first Kallippic cycle = 16/17 Phaophi, year 454 of Nabunassar, (20/21 Dec. 295 BC), dawn observation (JDN = 1614029); 15 Elaphebolion of the same Kallippic year = 5/6 Tybi (9/10 Mar. 294 BC), evening observation (JDN = 1614107); 8 Anthesterion, year 47 of the first Kallippic cycle = 29/30 Hathyr, year 465 of Nabunassar (29/30 Jan. 283 BC), evening observation (JDN = 1618087); and 25 Pyanepsion, year 48 of the first Kallippic cycle = 7/8 Thoth, year 466 of Nabunassar (8/9 Nov. 283 BC), dawn observation (JDN = 1618370). Fotheringham then concluded that the first Kallippic month, Hekatombaion, of the first cycle began at the sunset following both the summer solstice and the mean new moon in the early morning on 28 June 330 BC (JDN = 1601069).[15] The first Kallippic year, therefore, fell in year 8 of the 6th Metonic cycle. Table 2 below gives the 940 months of the Kallippic calendar, including 59 pairs of 30-30 months whose numbers may be computed from equations 10 and 11 with $1 \le n \le 59$. The cause of the 1-day discrepancy between some of the computed and manually determined day-numbers in the month immediately following the Kallippic sets of 30-30 months is similar to the one under the Metonic calendar.

I should, nevertheless, point out that the four Timocharian observations do not have a unique Kallippic epoch. Table 3 below gives other dates in the first Kallippic cycle, beginning with 16 July 329 BC and ending with 3 May 295 BC. These too satisfy equations 2 and 3 above and thus yield the same month and day-numbers for Timocharis' observations as those reckoned from 28 June 330 BC. However, with the exception of the latter, the remaining dates have little or no astronomical and/or calendrical significance and are, therefore, superfluous.[16]

A final point concerns the 1-day discrepancy between those computed and manually counted current day-numbers that fall after the eliminated day in every hollow month of the Metonic and

[10] Fotheringham (1924), 386, argues that the dates of these eclipses fall between the inceptions of the Metonic and Kallippic cycles and so their Attic months must be Metonic. Yet these records could have reached the Greek astronomers years after their compilation and not necessarily within the period 432-330 BC. Cf. also Samuel (1972), 46, and Toomer (1984), 211-213, who concur with Fotheringham.
[11] Jones (2000), 151-152, suggests that these may be on the Kallippic reckoning.
[12] Toomer (1984), 452-453 and 541.
[13] The calendrical significance of these dates will be discussed in some detail in a forthcoming paper.
[14] Fotheringham (1924), 387-392. For Ptolemy's treatment of these observations in the Almagest cf. Toomer (1984), 334-338.
[15] Fotheringham (1924), 390-391; NASA's JPL HORIZONS Web-Interface at: http://ssd.jpl.nasa.gov/horizons.cgi#top gives 01:22-01:23 UT and 01:39 UT as the moments of the summer solstice and new moon, respectively, on 28 June 330 BC.
[16] A series of dates in the Almagest place the first year of the first Kallippic cycle in 330/29 BC. Cf. Fotheringham (1924), 388; Samuel (1972), 47.

Callippic cycles. According to Geminus (*Eisagoge*, 8.56), it was not always the 30th day of the month that was dropped.[17] Each omission therefore led to a difference of one in two successive day-numbers when they fell either side of an eliminated day in the same calendar month. The following example clarifies the discrepancy:

Example 4: Convert 6299 and 6301 elapsed days into the Metonic or Kallippic calendar:

Using the above equations 2 and 3, we get $M_c = 214$ and $D_c = 9$ for $D = 6299$. However, although the 2-day difference between 6299 and 6301 means we should expect $D_c = 11$ for the latter, we get $M_c = 214$ but $D_c = 12$ for $D = 6301$. In other words, for a difference of 2 days in the number of elapsed days, there is a 3-day difference between the corresponding current day-numbers. This is caused by the elimination of the 6300th day of the cycle which would have been the 10th day of the 214th month. It should, therefore, be kept in mind that a manual count of days from the beginning of the cycle does not always lead to the correct day-number in the calendar if the effect of the eliminated day in the current month is overlooked. In the above example, because both the Metonic and Kallippic months 214 lack day 10, the numbers associated with the remaining days of that month after its 10th day are one ahead of those obtained through a manual day count from the beginning of the cycle.

Regrettably, with one exception,[18] we have no Metonic or Kallippic date that shares its month with an omitted day to ascertain whether the ancient astronomers employed the methods briefly described here to convert a given number of elapsed days into their calendars. In any case, it is possible that a competent astronomer-mathematician like Hipparchus would have had little trouble locating the 110 and 441 eliminated days in the Metonic and Kallippic calendars respectively, using some simple-to-operate equations as those given above. It is, therefore, unlikely that Hipparchus would have neglected the impact of the eliminated days even if we concede that he did not exactly follow our prescribed equations.

[17] Manitius (1898), 122-123; Heath (1913), 294; Aujac (1975), 57.

[18] The Kallippic date 14 Dios of the second Chaldean record in the Almagest, concerning the observation of Mercury at dawn on 30 Oct. 237 BC (JDN = 1635162), falls after the 430th eliminated day of the cycle ($N_e = 430$ from Equ. 12)) which removed the 10th day ($P_e = 10$ from Equ. 13) of the 918th calendar month. Its day-number 14 is, therefore, one more than that obtained from a direct count of days, starting with the inception of the Kallippic version of the Seleuco-Babylonian calendar on 1 Hyperberetaios = 27 August 311 BC. As explained earlier, because the day-count in the 918th Kallippic month goes from 9 to 11, missing the 10th day, the correct Kallippic day of observation would be the 14th day of the 918th month (= Dios). Cf. also Jones (2000), 147-148, giving a series of Kallippic dates in the period 28 Dec. 85 BC – 3(?) Jun. 74 BC. These carry Egyptian months and days, not Kallippic, and so cannot identify the eliminated days of the corresponding Kallippic months.

Table 1 – The Full and Hollow Metonic Months
(19-year Cycle begins with 13 Skirophorion. First complete month = Hecatombaion)

A	**Month-number**													**B**	**C**	**D**
1	1	2^{h}	3	4^{h}	5	6^{h}	7	8^{h}	9	10	11^{h}	12		5	355	355
2*	13^{h}	14	15^{h}	16	17^{h}	18	19^{h}	20	21^{h}	22	23^{h}	24	25^{h}	11	383	738
3	26	27	28^{h}	29	30^{h}	31	32^{h}	33	34^{h}	35	36^{h}	37		17	355	1093
4	38^{h}	39	40^{h}	41	42	43^{h}	44	45^{h}	46	47^{h}	48	49^{h}		23	354	1447
5*	50	51^{h}	52	53^{h}	54	55^{h}	56	57^{h}	58	59	60^{h}	61	62^{h}	29	384	1831
6	63	64^{h}	65	66^{h}	67	68^{h}	69	70^{h}	71	72^{h}	73	74		34	355	2186
7	75^{h}	76	77^{h}	78	79^{h}	80	81^{h}	82	83^{h}	84	85^{h}	86		40	354	2540
8*	87^{h}	88	89^{h}	90	91	92^{h}	93	94^{h}	95	96^{h}	97	98^{h}	99	46	384	2924
9	100^{h}	101	102^{h}	103	104^{h}	105	106	107^{h}	108	109^{h}	110	111^{h}		52	354	3278
10*	112	113^{h}	114	115^{h}	116	117^{h}	118	119^{h}	120	121^{h}	122	123	124^{h}	58	384	3662
11	125	126^{h}	127	128^{h}	129	130^{h}	131	132^{h}	133	134^{h}	135	136^{h}		64	354	4016
12	137	138	139^{h}	140	141^{h}	142	143^{h}	144	145^{h}	146	147^{h}	148		69	355	4371
13*	149^{h}	150	151^{h}	152	153^{h}	154	155	156^{h}	157	158^{h}	159	160^{h}	161	75	384	4755
14	162^{h}	163	164^{h}	165	166^{h}	167	168^{h}	169	170	171^{h}	172	173^{h}		81	354	5109
15	174	175^{h}	176	177^{h}	178	179^{h}	180	181^{h}	182	183^{h}	184	185^{h}		86	354	5463
16*	186	187	188^{h}	189	190^{h}	191	192^{h}	193	194^{h}	195	196^{h}	197	198^{h}	93	384	5847
17	199	200^{h}	201	202	203^{h}	204	205^{h}	206	207^{h}	208	209^{h}	210		98	355	6202
18*	211^{h}	212	213^{h}	214	215^{h}	216	217^{h}	218	219	220^{h}	221	222^{h}	223	104	384	6586
19	224^{h}	225	226^{h}	227	228^{h}	229	230^{h}	231	232^{h}	233	234	235^{h}		110	354	6940

<u>Key to Tables 1:</u>
A = year of the cycle
B = cumulative number of omitted days
C = days in year
D = cumulative number of days
* indicates embolismic year
^{h} indicates hollow month

Table 2 – The Full and Hollow Kallippic Months (76-year Cycle begins with 1 Hekatombaion)

A	Month-number													B	C	D
1*	1	2	3^h	4	5^h	6	7^h	8	9^h	10	11^h	12	13^h	6	384	384
2	14	15^h	16	17	18^h	19	20^h	21	22^h	23	24^h	25		11	355	739
3*	26^h	27	28^h	29	30^h	31	32^h	33	34	35^h	36	37^h	38	17	384	1123
4	39^h	40	41^h	42	43^h	44	45^h	46	47^h	48	49	50^h		23	354	1477
5	51	52^h	53	54^h	55	56^h	57	58^h	59	60^h	61	62^h		29	354	1831
6*	63	64^h	65	66	67^h	68	69^h	70	71^h	72	73^h	74	75^h	35	384	2215
7	76	77^h	78	79^h	80	81	82^h	83	84^h	85	86^h	87		40	355	2570
8	88^h	89	90^h	91	92^h	93	94^h	95	96^h	97	98	99^h		46	354	2924
9*	100	101^h	102	103^h	104	105^h	106	107^h	108	109^h	110	111^h	112	52	384	3308
10	113	114^h	115	116^h	117	118^h	119	120^h	121	122^h	123	124^h		58	354	3662
11*	125	126^h	127	128^h	129	130	131^h	132	133^h	134	135^h	136	137^h	64	384	4046
12	138	139^h	140	141^h	142	143^h	144	145	146^h	147	148^h	149		69	355	4401
13	150^h	151	152^h	153	154^h	155	156^h	157	158^h	159	160^h	161		75	354	4755
14*	162	163^h	164	165^h	166	167^h	168	169^h	170	171^h	172	173^h	174	81	384	5139
15	175^h	176	177	178^h	179	180^h	181	182^h	183	184^h	185	186^h		87	354	5493
16	187	188^h	189	190^h	191	192^h	193	194	195^h	196	197^h	198		92	355	5848
17*	199^h	200	201^h	202	203^h	204	205^h	206	207^h	208	209	210^h	211	98	384	6232
18	212^h	213	214^h	215	216^h	217	218^h	219	220^h	221	222^h	223		104	354	6586
19	224^h	225	226	227^h	228	229^h	230	231^h	232	233^h	234	235^h		110	354	6940
20*	236	237^h	238	239^h	240	241	242^h	243	244^h	245	246^h	247	248^h	116	384	7324
21	249	250^h	251	252^h	253	254^h	255	256^h	257	258	259^h	260		121	355	7679
22*	261^h	262	263^h	264	265^h	266	267^h	268	269^h	270	271^h	272	273	127	384	8063
23	274^h	275	276^h	277	278^h	279	280^h	281	282^h	283	284^h	285		133	354	8417
24	286^h	287	288^h	289	290	291^h	292	293^h	294	295^h	296	297^h		139	354	8771
25*	298	299^h	300	301^h	302	303^h	304	305	306^h	307	308^h	309	310^h	145	384	9155
26	311	312^h	313	314^h	315	316^h	317	318^h	319	320^h	321	322		150	355	9510
27	323^h	324	325^h	326	327^h	328	329^h	330	331^h	332	333^h	334		156	354	9864
28*	335^h	336	337	338^h	339	340^h	341	342^h	343	344^h	345	346^h	347	162	384	10248
29	348^h	349	350^h	351	352^h	353	354	355^h	356	357^h	358	359^h		168	354	10602
30*	360	361^h	362	363^h	364	365^h	366	367^h	368	369	370^h	371	372^h	174	384	10986
31	373	374^h	375	376^h	377	378^h	379	380^h	381	382^h	383	384^h		180	354	11340
32	385	386	387^h	388	389^h	390	391^h	392	393^h	394	395^h	396		185	355	11695
33*	397^h	398	399^h	400	401	402^h	403	404^h	405	406^h	407	408^h	409	191	384	12079
34	410^h	411	412^h	413	414^h	415	416^h	417	418	419^h	420	421^h		197	354	12433
35	422	423^h	424	425^h	426	427^h	428	429^h	430	431^h	432	433		202	355	12788
36*	434^h	435	436^h	437	438^h	439	440^h	441	442^h	443	444^h	445	446^h	209	383	13171
37	447	448^h	449	450	451^h	452	453^h	454	455^h	456	457^h	458		214	355	13526
38	459^h	460	461^h	462	463^h	464	465	466^h	467	468^h	469	470^h		220	354	13880
39*	471	472^h	473	474^h	475	476^h	477	478^h	479	480^h	481	482	483^h	226	384	14264
40	484	485^h	486	487^h	488	489^h	490	491^h	492	493^h	494	495^h		232	354	14618
41*	496	497	498^h	499	500^h	501	502^h	503	504^h	505	506^h	507	508^h	238	384	15002
42	509	510^h	511	512^h	513	514	515^h	516	517^h	518	519^h	520		243	355	15357
43	521^h	522	523^h	524	525^h	526	527^h	528	529	530^h	531	532^h		249	354	15711
44*	533	534^h	535	536^h	537	538^h	539	540^h	541	542^h	543	544^h	545	255	384	16095
45	546	547^h	548	549^h	550	551^h	552	553^h	554	555^h	556	557^h		261	354	16449
46	558	559^h	560	561	562^h	563	564^h	565	566^h	567	568^h	569		266	355	16804
47*	570^h	571	572^h	573	574^h	575	576^h	577	578	579^h	580	581^h	582	272	384	17188
48	583^h	584	585^h	586	587^h	588	589^h	590	591^h	592	593	594^h		278	354	17542
49*	595	596^h	597	598^h	599	600^h	601	602^h	603	604^h	605	606^h	607	284	384	17926
50	608^h	609	610	611^h	612	613^h	614	615^h	616	617^h	618	619^h		290	354	18280
51	620	621^h	622	623^h	624	625	626^h	627	628^h	629	630^h	631		295	355	18635
52*	632^h	633	634^h	635	636^h	637	638^h	639	640^h	641	642	643^h	644	301	384	19019
53	645^h	646	647^h	648	649^h	650	651^h	652	653^h	654	655^h	656		307	354	19373
54	657	658^h	659	660^h	661	662^h	663	664^h	665	666^h	667	668^h		313	354	19727
55*	669	670^h	671	672^h	673	674	675^h	676	677^h	678	679^h	680	681^h	319	384	20111
56	682	683^h	684	685^h	686	687^h	688	689	690^h	691	692^h	693		324	355	20466
57	694^h	695	696^h	697	698^h	699	700^h	701	702^h	703	704^h	705		330	354	20820

Table 2 – Continued

A	Month-number													B	C	D
58*	706	707^{h}	708	709^{h}	710	711^{h}	712	713^{h}	714	715^{h}	716	717^{h}	718	336	384	21204
59	719^{h}	720	721	722^{h}	723	724^{h}	725	726^{h}	727	728^{h}	729	730^{h}		342	354	21558
60*	731	732^{h}	733	734^{h}	735	736^{h}	737	738	739^{h}	740	741^{h}	742	743^{h}	348	384	21942
61	744	745^{h}	746	747^{h}	748	749^{h}	750	751^{h}	752	753	754^{h}	755		353	355	22297
62	756^{h}	757	758^{h}	759	760^{h}	761	762^{h}	763	764^{h}	765	766^{h}	767		359	354	22651
63*	768^{h}	769	770	771^{h}	772	773^{h}	774	775^{h}	776	777^{h}	778	779^{h}	780	365	384	23035
64	781^{h}	782	783^{h}	784	785	786^{h}	787	788^{h}	789	790^{h}	791	792^{h}		371	354	23389
65	793	794^{h}	795	796^{h}	797	798^{h}	799	800^{h}	801	802	803^{h}	804		376	355	23744
66*	805^{h}	806	807^{h}	808	809^{h}	810	811^{h}	812	813^{h}	814	815^{h}	816	817	382	384	24128
67	818^{h}	819	820^{h}	821	822^{h}	823	824^{h}	825	826^{h}	827	828^{h}	829		388	354	24482
68*	830^{h}	831	832^{h}	833	834	835^{h}	836	837^{h}	838	839^{h}	840	841^{h}	842	394	384	24866
69	843^{h}	844	845^{h}	846	847^{h}	848	849	850^{h}	851	852^{h}	853	854^{h}		400	354	25220
70	855	856^{h}	857	858^{h}	859	860^{h}	861	862^{h}	863	864^{h}	865	866		405	355	25575
71*	867^{h}	868	869^{h}	870	871^{h}	872	873^{h}	874	875^{h}	876	877^{h}	878	879^{h}	412	383	25958
72	880	881	882^{h}	883	884^{h}	885	886^{h}	887	888^{h}	889	890^{h}	891		417	355	26313
73	892^{h}	893	894^{h}	895	896^{h}	897	898	899^{h}	900	901^{h}	902	903^{h}		423	354	26667
74*	904	905^{h}	906	907^{h}	908	909^{h}	910	911^{h}	912	913	914^{h}	915	916^{h}	429	384	27051
75	917	918^{h}	919	920^{h}	921	922^{h}	923	924^{h}	925	926^{h}	927	928^{h}		435	354	27405
76	929	930	931^{h}	932	933^{h}	934	935^{h}	936	937^{h}	938	939^{h}	940^{h}		441	354	27759

Key to Table 2:

A = year of the cycle
B = cumulative number of omitted days
C = days in year
D = cumulative number of days
* indicates embolismic year
^{h} indicates hollow month

Table 3 – Possible Kallippic Epochs of the Four Timocharian Observations

No.	Date (BC)	JDN	No.	Date (BC)	JDN	No.	Date (BC)	JDN
1	28 Jun. 330	1601069	24	7 Feb. 317	1605676	47	29 Oct. 306	1609958
2	16 Jul. 329	1601453	25	6 Apr. 317	1605735	48	27 Dec. 306	1610017
3	13 Sep. 329	1601512	26	4 Jun. 317	1605794	49	14 Jan. 304	1610401
4	2 Oct. 328	1601896	27	23 Jan. 316	1606178	50	14 Mar. 304	1610460
5	30 Nov. 328	1601955	28	21 Aug. 316	1606237	51	12 May 304	1610519
6	28 Jan. 327	1602014	29	9 Sep. 315	1606621	52	31 May 303	1610903
7	16 Feb. 326	1602398	30	7 Nov. 315	1606680	53	29 Jul. 303	1610962
8	16 Apr. 326	1602457	31	5 Jan. 314	1606739	54	17 Aug. 302	1611346
9	4 May 325	1602841	32	24 Jan. 313	1607123	55	15 Oct. 302	1611405
10	2 Jul. 325	1602900	33	23 Mar. 313	1607182	56	13 Dec. 302	1611464
11	30 Aug. 325	1602959	34	11 Apr. 312	1607566	57	31 Dec. 301	1611848
12	18 Sep. 324	1603343	35	9 Jan. 312	1607625	58	28 Feb. 300	1611907
13	16 Nov. 324	1603402	36	7 Aug. 312	1607684	59	19 Mar. 299	1612291
14	5 Dec. 323	1603786	37	26 Aug. 311	1608068	60	17 May 299	1612350
15	2 Feb. 322	1603845	38	24 Oct. 311	1608127	61	15 Jul. 299	1612409
16	2 Apr. 322	1603904	39	12 Nov. 310	1608511	62	3 Aug. 298	1612793
17	20 Apr. 321	1604288	40	10 Jan. 309	1608570	63	1 Oct. 298	1612852
18	18 Jan. 321	1604347	41	9 Mar. 309	1608629	64	19 Oct. 297	1613236
19	7 Jul. 320	1604731	42	28 Mar. 308	1609013	65	17 Dec. 297	1613295
20	4 Sep. 320	1604790	43	26 May 308	1609072	66	14 Feb. 296	1613354
21	2 Nov. 320	1604849	44	14 Jun. 307	1609456	67	5 Mar. 295	1613738
22	21 Nov. 319	1605233	45	12 Aug. 307	1609515	68	3 May 295	1613797
23	19 Jan. 318	1605292	46	10 Oct. 307	1609574			

Abbreviations:

SEB Seleucid Era of the Babylonian calendar (epoch = 2/3 April 311 BC = 1 Nīsānu)
JDN Julian Day Number

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1- Adjusted page 1 on 3 October 2008: I've formulated Van der Waerden's procedure.
2- Adjusted page 6 on 3 October 2008: Abbreviations: JDN = Julian Day Number.
3- Added footnote 16 on 3 October 2008.
4- Modified on 4-5 May 2013 some paragraphs concerning the 1-day discrepancy between the calendrical and manual counts, caused by the position of the eliminated day after each set of 30-30 months.